field intensities of E = 636, 673, 730, 795, and 860 V/m were found from the experimentally determined volt–ampere curve [1] for fixed current values of I = 30, 50, 70, 90, and 110 A.

The results of calculation of the temperature distribution T(r), presented in Fig. 4, are close to those of a numerical solution by the step approximation method. The following values of approximation parameters were used: $A = 6342\ (\Omega\bullet\text{m})^{-1}$, B = 7797 W/m, and $S_c = 8292$ W/m. These values ensure a good approximation of the function σ(S) for S values not exceeding 8000 W/m, which corresponds to a temperature of $T \simeq 15{,}000°K$.

In conclusion, it should be noted that the proposed method, being an analog of the coarse linearization method (quasichannel model), has the advantage that calculation of auxiliary parameters is significantly simplified.

NOTATION

T, temperature; σ, electrical conductivity; r, radial coordinate; d = 2R, diameter of discharge channel; λ, thermal conductivity; S, thermal conductivity function; p, pressure; E, electric field intensity; I, current; A, B, S_c, parameters of approximation ellipse; G, gas expenditure; u, auxiliary variable; f, auxiliary function. Indices: b, boundary; *a*, axial.

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HEAT AND MOISTURE TRANSFER BETWEEN A FRESHLY EXPOSED ROCK MASS AND A VENTILATING AIR JET

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The problem of heat and moisture transfer between an infinite isotropic rock mass and a ventilating air jet of constant temperature is considered. Equations are derived for the temperature- and moisture-transfer potential fields. Nomograms are presented for calculating the heat and moisture flows.

Coal mines are now sunk to depths of 1000-1100 m. In view of the current increase in the mining of coking coals the working depth is likely to increase still further. In order to create normal labor conditions in deep shafts it is essential to introduce a system of air-cooling and to improve methods of calculating the thermal characteristics of mines.

Existing methods of calculation [1, 2] are based on solving the problem of transient heat transfer between the ventilating jet and the rock mass surrounding the working.

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Many investigations have nevertheless shown that the air in the shafts acquires more moisture as a result of the drying of the rock mass, the heat- and moisture-transfer processes reaching their greatest intensity in freshly opened workings, when most of the moisture passes into the air by evaporation from the walls.

In this case it is reasonable to assume that the criterion of the phase transition in the rock mass is close to zero ($\varepsilon \approx 0$) and the differential equations of heat and moisture transfer between the rock mass and the jet will assume the form [3]

$$\begin{cases} \dfrac{\partial t}{\partial \tau} = a_q \left(\dfrac{\partial^2 t}{\partial R^2} + \dfrac{1}{R} \cdot \dfrac{\partial t}{\partial R} \right) \equiv a_q \nabla^2 t, \\ \dfrac{\partial \theta}{\partial \tau} = a_m \left(\dfrac{\partial^2 \theta}{\partial R^2} + \dfrac{1}{R} \cdot \dfrac{\partial \theta}{\partial R} \right) + a_m \delta \nabla^2 t \quad (R_0 \leqslant R) \end{cases} \tag{1}$$

with the following boundary conditions:

$$t(R,\ 0) = t_n,\ \theta(R,\ 0) = \theta_0, \tag{2}$$

$$\text{for} \quad R \to \infty \quad t(R,\ \tau) \to t_n,\ \theta(R,\ \tau) \to \theta_0, \tag{3}$$

$$-a_m \frac{\partial \theta(R_0,\ \tau)}{\partial R} - a_m \delta \frac{\partial t(R_0,\ \tau)}{\partial R} + \beta[\theta(R_0,\ \tau) - \theta_e] = 0, \tag{4}$$

$$-\beta\rho\gamma_0[\theta(R_0,\ \tau) - \theta_e] + \lambda \frac{\partial t(R_0,\ \tau)}{\partial R} + \alpha[t_j - t(R_0,\ \tau)] = 0. \tag{5}$$

We may introduce the reduced heat-transfer coefficient

$$\begin{gathered} \bar{\alpha} = \alpha_{\text{red}} = \alpha + \frac{\beta\rho\gamma_0[\theta(R_0,\ \tau) - \theta_e]}{t(R_0,\ \tau) - t_j} = \alpha(1 + l), \\ l = \frac{\beta\rho\gamma_0[\theta(R_0,\ \tau) - \theta_e]}{\alpha[t(R_0,\ \tau) - t_j]}, \end{gathered} \tag{6}$$

after which boundary condition (5) becomes

$$\lambda \frac{\partial t(R_0,\ \tau)}{\partial R} + \bar{\alpha}[t_j - t(R_0,\ \tau)] = 0. \tag{7}$$

We apply a Laplace–Carson transform [4] to the system (1):

$$\overline{f(R,\ p)} = F(R,\ p) = p \int_0^{+\infty} \exp(-p\tau) f(R,\ \tau)\, d\tau.$$

Then

$$\begin{cases} \dfrac{d^2 \bar{T}_1}{dR^2} + \dfrac{1}{R} \cdot \dfrac{d\bar{T}_1}{dR} = \dfrac{p}{a_q} \bar{T}_1, \\ \dfrac{d^2 \bar{\theta}_1}{dR^2} + \dfrac{1}{R} \cdot \dfrac{d\bar{\theta}_1}{dR} = \dfrac{p}{a_m} \bar{\theta}_1 - \dfrac{\delta}{a_q} p\bar{T}_1 \end{cases} \tag{8}$$

with the following boundary conditions:

$$\text{for} \quad R \to \infty \quad \bar{T}_1(R,\ p) \to 0,\ \bar{\theta}_1(R,\ p) \to 0, \tag{9}$$

$$-a_m \frac{d\bar{\theta}_1(R_0,\ p)}{dR} - a_m \delta \frac{d\bar{T}_1(R_0,\ p)}{dR} + \beta[\bar{\theta}_1(R_0,\ p) + \theta_0 - \theta_e] = 0, \tag{10}$$

$$\lambda \frac{d\bar{T}_1(R_0,\ p)}{dR} + \bar{\alpha}[t_j - t_p - \bar{T}_1(R_0,\ p)] = 0. \tag{11}$$

It is easy to show that

$$\bar{T}_1(R,\ p) = CK_0\left(R\sqrt{p/a_q}\right). \tag{12}$$

Substituting (12) into (8) gives

$$\bar{\theta}_1(R,\ p) = dCK_0\left(R\sqrt{p/a_q}\right) + MK_0\left(R\sqrt{p/a_m}\right),\ d = \delta/(a_q/a_m - 1). \tag{13}$$

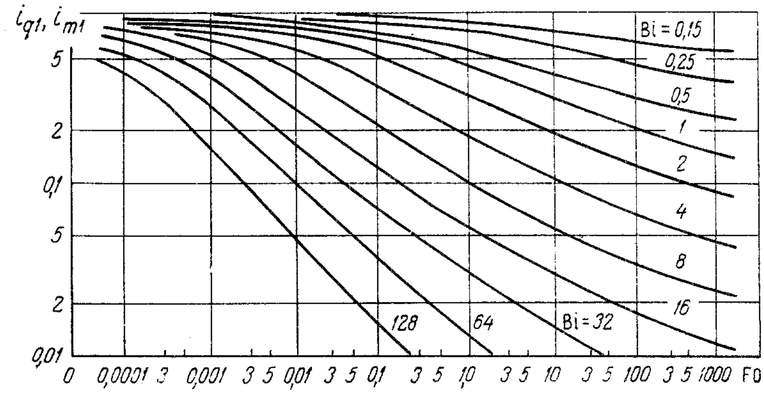

Fig. 1. Nomogram of the functions i_{q1} and i_{m1}.

After using boundary conditions (10) and (11) we find the image of the rock-mass temperature and moisture-transfer potential:

$$\bar{T}(R,\ p) = \frac{\bar{t} - t_p}{t_j - t_p} = \frac{K_0\left(R\sqrt{p/a_q}\right)}{K_0\left(R_0\sqrt{p/a_q}\right) + \frac{\lambda\sqrt{p}}{\alpha\sqrt{a_q}}\,K_1\left(R_0\sqrt{p/a_q}\right)}\,, \tag{14}$$

$$\bar{\vartheta}(R,\ p) = \frac{\bar{\theta} - \theta_0}{\theta_e - \theta_0} = \frac{[1 + k(t_p - t_j)]\,K_0\left(R\sqrt{p/a_m}\right)}{K_0\left(R_0\sqrt{p/a_m}\right) + \frac{\sqrt{a_m}}{\beta}\sqrt{p}\,K_1\left(R_0\sqrt{p/a_m}\right)} +$$

$$+ \frac{d(t_p - t_j)\,K_0\left(R\sqrt{p/a_q}\right)}{(\theta_0 - \theta_e)\left[K_0\left(R_0\sqrt{p/a_q}\right) + \frac{\lambda}{\alpha\sqrt{a_q}}\sqrt{p}\,K_1\left(R_0\sqrt{p/a_q}\right)\right]} +$$

$$+ \frac{(d - k)(t_p - t_j)\,K_0\left(R_0\sqrt{p/a_q}\right)K_0\left(R\sqrt{p/a_m}\right)}{\left[K_0\left(R_0\sqrt{p/a_q}\right) + \frac{\lambda\sqrt{p}}{\alpha\sqrt{a_q}}\,K_1\left(R_0\sqrt{p/a_q}\right)\right]\left[K_0\left(R_0\sqrt{p/a_m}\right) + \frac{\sqrt{a_m p}}{\beta}\,K_1\left(R_0\sqrt{p/a_m}\right)\right]} \tag{15}$$

with notation

$$k = a_m(d + \delta)\,\bar{\alpha}/\beta\lambda;\quad d = \delta/(a_q/a_m - 1). \tag{16}$$

In order to find the original we use the inversion theorem, integrating the image around a special contour [6]:

$$U(z,\ \mathrm{Fo}_q,\ \mathrm{Bi}_q) = \frac{t - t_j}{t_p - t_j} = i_q = \frac{2}{\pi}\int\limits_0^{+\infty} \frac{\left\{N_0(\mu z)\left[J_0(\mu) + \frac{\mu}{\mathrm{Bi}_q}J_1(\mu)\right] - J_0(\mu z)\left[N_0(\mu) + \frac{\mu}{\mathrm{Bi}_q}N_1(\mu)\right]\right\}\exp(-\mu^2\mathrm{Fo}_q)\,d\mu}{\mu\left\{\left[J_0(\mu) + \frac{\mu}{\mathrm{Bi}_q}J_1(\mu)\right]^2 + \left[N_0(\mu) + \frac{\mu}{\mathrm{Bi}_q}N_1(\mu)\right]^2\right\}}\,, \tag{17}$$

$$V(z,\ \mathrm{Fo}_q,\ \mathrm{Fo}_m,\ \mathrm{Bi}_q,\ \mathrm{Bi}_m) = \frac{\theta - \theta_e}{\theta_0 - \theta_e} = i_m - k(t_p - t_j)(1 - i_m) -$$

$$- \frac{d'(t_p - t_j)}{\theta_0 - \theta_e}(1 - i_q) - (d - k)(t_p - t_j)(1 + i_2) = [1 - k(t_p - t_j)] \times$$

$$\times \frac{2}{\pi}\int\limits_0^{+\infty} \frac{\left\{N_0(\mu z)\left[J_0(\mu) + \frac{\mu}{\mathrm{Bi}_m}J_1(\mu)\right] - J_0(\mu z)\left[N_0(\mu) + \frac{\mu}{\mathrm{Bi}_m}N_1(\mu)\right]\right\}\exp(-\mu^2\mathrm{Fo}_m)\,d\mu}{\mu\left\{\left[J_0(\mu) + \frac{\mu}{\mathrm{Bi}_m}J_1(\mu)\right]^2 + \left[N_0(\mu) + \frac{\mu}{\mathrm{Bi}_m}N_1(\mu)\right]^2\right\}}$$

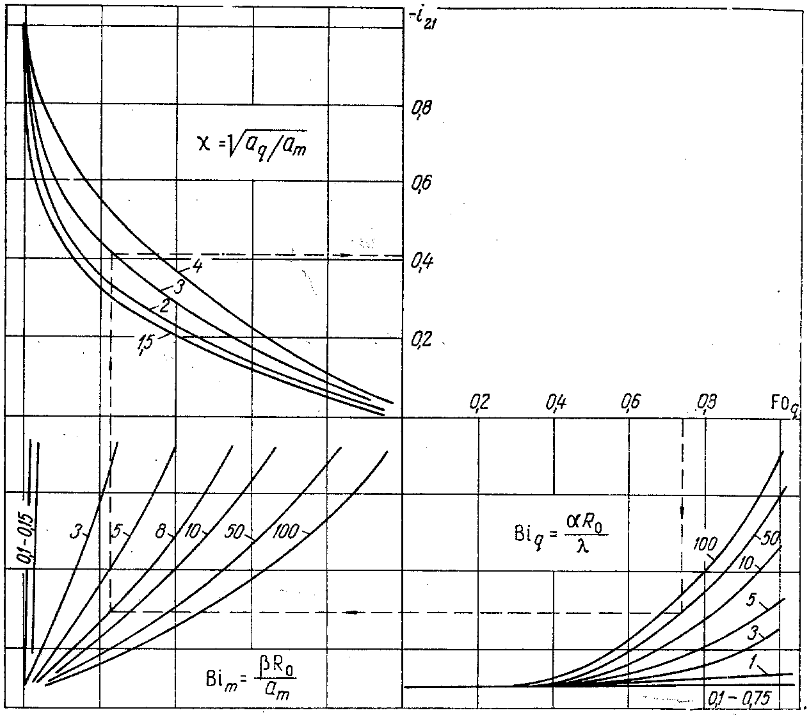

Fig. 2. Nomogram of the functions $i_{21}(Fo_q, Bi_q, Bi_m, \chi)$ and $-i_{21} = f(Fo_q, Bi_q, Bi_m, \chi)$; $Fo_q = a_q\tau/R_0^2$.

$$+\frac{d(t_p - t_j)}{\theta_0 - \theta_e}\cdot\frac{2}{\pi}\int_0^{+\infty}\frac{\left\{N_0(\mu z)\left[J_0(\mu)+\frac{\mu}{Bi_q}J_1(\mu)\right] - J_0(\mu z)\left[N_0(\mu)+\frac{\mu}{Bi_q}N_1(\mu)\right]\right\}\exp(-\mu^2 Fo_q)\,d\mu}{\mu\left\{\left[J_0(\mu)+\frac{\mu}{Bi_q}J_1(\mu)\right]^2+\left[N_0(\mu)+\frac{\mu}{Bi_q}N_1(\mu)\right]^2\right\}} - \frac{2(d-k)(t_p-t_j)}{\pi}\times$$

$$\times\int_0^{+\infty}\frac{(AE-BD)\times\exp(-\mu^2 Fo_q)\,d\mu/\mu}{\left\{\left[J_0(\mu)+\frac{\mu}{Bi_q}J_1(\mu)\right]^2+\left[N_0(\mu)+\frac{\mu}{Bi_q}N_1(\mu)\right]^2\right\}\times\left\{\left[J_0(\mu\chi)+\frac{\mu\chi}{Bi_m}J_1(\mu\chi)\right]^2+\left[N_0(\mu\chi)+\frac{\mu\chi}{Bi_m}N_1(\mu\chi)\right]^2\right\}}, \quad (18)$$

where

$$Bi_q = \bar{\alpha}R_0/\lambda,\ Bi_m = \beta R_0/a_m,$$
$$Fo_q = a_q\tau/R_0^2,\ Fo_m = a_m\tau/R_0^2,\ \chi = \sqrt{a_q/a_m},\ z = R/R_0,$$

and A, B, D, E are certain rational functions of Bessel functions. Equations (17) and (18) express the dimensionless temperature of the rock mass and the dimensionless moisture-transfer potential in the latter.

For calculating the heat and moisture flows we introduce two dimensionless complexes. Following [2] we determine the dimensionless transient heat-transfer coefficient:

$$\mathrm{Ku}_\tau = \frac{R_0}{\lambda}\, k_\tau = \frac{R_0}{t_p - t_j} \cdot \left.\frac{\partial t}{\partial R}\right|_{R=R_0} = \mathrm{Bi}_q U|_{z=1} = \mathrm{Bi}_q i_q|_{z=1} = \mathrm{Bi}_q i_{q1} = \frac{4}{\pi^2} \times$$

$$\times \int_0^{+\infty} \frac{\exp(-\mu^2 \mathrm{Fo}_q)\, d\mu}{\mu \left\{ \left[J_0(\mu) + \frac{\mu}{\mathrm{Bi}_q} J_1(\mu) \right]^2 + \left[N_0(\mu) + \frac{\mu}{\mathrm{Bi}_q} N_1(\mu) \right]^2 \right\}}. \quad (19)$$

In an analogous way we may determine the dimensionless coefficient of transient mass transfer:

$$\mu_\tau = \frac{R_0}{a_m}\, m_\tau = \frac{R_0}{\theta_0 - \theta_e} \cdot \left.\frac{\partial \theta}{\partial R}\right|_{R=R_0} = \mathrm{Bi}_m V|_{z=1} =$$

$$= \mathrm{Bi}_m [1 - k(t_p - t_j)]\, i_{m1} + \mathrm{Bi}_m d(t_p - t_j)\, i_{q1}/(\theta_0 - \theta_e) - \mathrm{Bi}_m (d - k)(t_p - t_j)\, i_{21}, \quad (20)$$

where the coefficients d and k are introduced by Eq. (16); the function $\mathrm{Bi}_m i_{m1}(\mathrm{Bi}_m, \mathrm{Fo}_m)$ has a form analogous to that of Eq. (19), with the replacement of Fo_q and Bi_q by Fo_m, Bi_m

$$i_{21} = i_2|_{z=1} = \frac{2}{\pi} \int_0^{+\infty} \frac{(A_1E_1 - B_1D_1) \times}{\left\{ \left[J_0(\mu) + \frac{\mu}{\mathrm{Bi}_q} J_1(\mu) \right]^2 + \left[N_0(\mu) + \frac{\mu}{\mathrm{Bi}_q} N_1(\mu) \right]^2 \right\} \times} \rightarrow$$

$$\rightarrow \frac{\times \exp(-\mu^2 \mathrm{Fo}_q)\, d\mu/\mu}{\times \left\{ \left[J_0(\mu\chi) + \frac{\mu\chi}{\mathrm{Bi}_m} J_1(\mu\chi) \right]^2 + \left[N_0(\mu\chi) + \frac{\mu\chi}{\mathrm{Bi}_m} N_1(\mu\chi) \right]^2 \right\}}. \quad (21)$$

The integrals i_{q1}, i_{m1}, and i_{21} in the computing equations (19) and (20) were calculated on an electronic computer over wide ranges of the parameters Fo_q, Fo_m, Bi_q, Bi_m, and $\chi = \sqrt{a_q/a_m}$. The nomograms presented (Figs. 1 and 2) enable us to calculate the heat and moisture flows in any particular case by means of the equations

$$q = k_\tau (t_p - t_j), \quad (22)$$

$$m = m_\tau (\theta_0 - \theta_e). \quad (23)$$

NOTATION

R, cylindrical coordinate perpendicular to the axis of the working; R_0, radius of the cylindrical working; $t(R, \tau)$, temperature of the rock mass; $\theta(R, \tau)$, moisture-transfer potential; c_q, c_T', specific heat and moisture capacity, respectively; $a_q = \lambda/c_q\gamma_0$, thermal diffusivity; $a_m = \lambda_m/c_T'\gamma_0$, moisture-transfer potential conductivity; λ, thermal conductivity; λ_m, moisture conductivity; δ, thermal-gradient coefficient; ρ, specific heat of phase transition; ε, phase-transition criterion ($0 \le \varepsilon \le 1$); α, β, heat- and mass-transfer coefficients; t_j, jet temperature; θ_e, equilibrium value of the mass-transfer potential; $J_0(x)$, $J_1(x)$, $N_0(x)$, $N_1(x)$, Bessel functions of the first and second kinds and of the zero and first orders [5]; $K_0(x)$, $K_1(x)$, modified Hankel functions of the zero and first orders [5].

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